Level statistics of quantum dots coupled to reservoirs

Jürgen König[1], Yuval Gefen[2], and Gerd Schön[1]
[1] *Institut für Theoretische Festkörperphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany*
[2] *Department of Condensed Matter Physics, The Weizmann Institute of Science, 76100 Rehovot, Israel*
(February 1, 2008)

We study the effect of electron tunneling on the level statistics of quantum dots. While the coupling between individual levels and the electron reservoir leads predominantly to the expected level broadening, the indirect coupling of adjacent levels via the reservoir results in a new asymptotic level statistics and broadening. These features, which are attributed to renormalized spectral weights rather than renormalized eigenvalues of the Hamiltonian, are observable in the asymptotic frequency dependence in an absorption experiment.

Introduction. The statistics of the excitation spectra determines many physical properties of quantum dots. These include thermodynamic properties, transport coefficients, the response to external a.c. fields, and statistical fluctuations [1–5]. For a model of non-interacting electrons confined within a moderately disordered quantum dot (diffusive disorder), the description of the single-particle levels for energies below the Thouless energy conforms to the Wigner-Dyson theory of random matrices (RMT) [3,6–8]. As a result of the level repulsion, on energy scales smaller than the average single-particle spacing Δ, the statistics of the interlevel spacings for $s \ll 1$ is given by $P_o(s) = \pi s/2$, if the random system possesses time reversal symmetry, (orthogonal case) and by $P_u(s) = 32s^2/\pi^2$ otherwise (unitary case) [9]. Here s is measured in units of Δ. This level statistics manifests itself in the response to low-frequency fields. For instance, the a.c. dissipative conductance scales as the applied frequency ω_0, or as ω_0^2, for the two symmetry classes, respectively. This result, valid for the grandcanonical ensemble, is modified for the canonical ensemble [1,2,5,10,11].

If the quantum dot with discrete bare levels is coupled to external degrees of freedom, these levels acquire a finite width Γ. While the broadening is known to depend on the coupling and on the energy measured from ϵ_F (see e.g. [12]) it is commonly assumed that the levels statistics is unchanged. This means, the standard Kubo-Greenwood formalism can be employed, provided that the δ-function describing the energy conservation is replaced by a Lorentzian with finite width.

In this Letter we study the influence of a dot-reservoir coupling on the effective level statistics and the asymptotic ω_0-dependence of the absorbed power. A simple, though generic, model, where the quantum dot is coupled to an ideal reservoir of non-interacting electrons is used. In most of our analysis we ignore interactions in the dot. Breaking the RMT invariance by the choice of a preferred basis may modify the level statistics as it was demonstrated for disorder in metals in Ref. [13]. In our case, the system is not invariant under rotation between the dot and the lead states. We show that the coupling to the lead gives rise to a renormalization of both the low-frequency spectrum and the bare level broadening Γ. *We attribute the origin of the modified statistics and broadening (Eqs. (8)-(10)) to renormalized spectral weights rather than renormalized eigenvalues of the Hamiltonian.* Subsequently the frequency- and temperature-dependent absorption is calculated, (Eqs. (12) with (20)-(23)). Finally we comment briefly on the effect of Coulomb interactions.

Model. The model Hamiltonian

$$H = \sum_k \epsilon_k a_k^\dagger a_k + \sum_{\sigma=1,2} \epsilon_\sigma c_\sigma^\dagger c_\sigma + \sum_{k\sigma} \left[T_{k\sigma} a_k^\dagger c_\sigma + h.c. \right] \quad (1)$$

describes a quantum dot with two levels $\sigma = 1, 2$ coupled to the *same* electron reservoir [14]. The tunnel matrix elements are assumed to be independent of the reservoir state, $T_{k\sigma} = T_\sigma$. The Coulomb interaction will be discussed at the end this Letter.

Due to the coupling to the reservoirs, if both dot levels are well separated, each level acquires a finite width related to $\Gamma_\sigma(\omega) = 2\pi \sum_k |T_{k\sigma}|^2 \delta(\omega - \epsilon_k)$. However, if the levels are degenerate, $\epsilon_1 = \epsilon_2 = \epsilon$, a rotation within the dot subspace yields

$$H' = \sum_k \epsilon_k a_k^\dagger a_k + \epsilon \sum_{\sigma=1,2} c'^\dagger_\sigma c'_\sigma + \sum_k \left[T_{k1} a_k^\dagger c'_1 + h.c. \right] \quad (2)$$

where $c'_1 = (T_1 c_1 + T_2 c_2)/\sqrt{|T_1|^2 + |T_2|^2}$ and $c'_2 = (T_2 c_1 - T_1 c_2)/\sqrt{|T_1|^2 + |T_2|^2}$. In this form one recognizes that only the new level 1 is coupled to a reservoir of electrons (leading to a Lorentzian form of the spectral density) while the new level 2 is isolated (showing therefore a δ-peaked spectral density). Therefore, in the unrotated basis c_1, c_2, the spectral density of each level consists of a Lorentzian part but also contains a δ-function. The sharp peak persists if the degeneracy is slightly lifted.

Spectral density. In order to catch the evolution of the sharp peak in the spectral density quantitatively, we employ the equations of motion for the Green's functions $G_{\sigma\sigma}(t) := -i\Theta(t)\langle\{c_\sigma(t), c_\sigma(0)\}\rangle$. Since the Hamiltonian Eq. (1) is quadratic, we can find the exact solution

$$G_{\sigma\sigma}(\omega) = \left[\omega - \epsilon_\sigma - \Sigma_{\sigma\sigma} - \frac{\Sigma_{\sigma\bar\sigma}\Sigma_{\bar\sigma\sigma}}{\omega - \epsilon_{\bar\sigma} - \Sigma_{\bar\sigma\bar\sigma}} \right]^{-1} \quad (3)$$

with $\Sigma_{\sigma\sigma'} = \sum_k T_{k\sigma} T^*_{k\sigma'}/(\omega - \epsilon_k)$, i.e. $\Sigma_{\sigma\sigma}(\omega + i0^+) = -i\Gamma_\sigma/2$ and $\Sigma_{\sigma\bar\sigma}(\omega + i0^+)\Sigma_{\bar\sigma\sigma}(\omega + i0^+) = -\Gamma_\sigma\Gamma_{\bar\sigma}/4$ (for

simplicity we put $\hbar = 1$). The spectral function follows from $A_\sigma(\omega) = -\mathrm{Im}\, G_{\sigma\sigma}(\omega + i0^+)/\pi$. For $\epsilon_1 \neq \epsilon_2$ it is

$$A_\sigma(\omega) = \frac{\Gamma_\sigma \left(\omega - \epsilon_{\bar\sigma}\right)^2 /2\pi}{\left(\omega - \epsilon_\sigma\right)^2 \left(\omega - \epsilon_{\bar\sigma}\right)^2 + \left[(\Gamma_1+\Gamma_2)\omega/4 - \hat\epsilon\right]^2} \quad (4)$$

with $\hat\epsilon = (\Gamma_1\epsilon_2 + \Gamma_2\epsilon_1)/(\Gamma_1+\Gamma_2)$. In Fig. 1 we plot the spectral density of level 2 for several ratios $\Gamma/\Delta\epsilon$. For the following we define $\Delta\epsilon = \epsilon_2 - \epsilon_1$. If the bare levels are well separated, i.e., $\Delta\epsilon \gg \Gamma$, the spectral density is approximately a Lorentzian with width Γ_σ,

$$A_\sigma(\omega) = \frac{1}{\pi}\frac{\Gamma_\sigma/2}{(\omega-\epsilon_\sigma)^2 + (\Gamma_\sigma/2)^2}, \quad (5)$$

and the maximum is at the bare value ϵ_σ, i.e., the level splitting remains unrenormalized $\Delta\tilde\epsilon = \Delta\epsilon$. We see, however, from Fig. 1 that things are different in the opposite limit $\Delta\epsilon \ll \Gamma$. Then, the level position is renormalized which leads to a modified level statistics, and the width is much smaller than the bare broadening.

For the degenerate case $\epsilon_1 = \epsilon_2$ we find

$$A_\sigma(\omega) = \frac{\Gamma_\sigma/2\pi}{\left(\omega-\epsilon_\sigma\right)^2 + (\Gamma_1+\Gamma_2)^2/4} + \frac{\Gamma_{\bar\sigma}\delta(\omega-\epsilon_\sigma)}{\Gamma_1+\Gamma_2} \quad (6)$$

and recover the expected δ peak.

Modified level statistics. For $\Delta\epsilon \ll \Gamma$, Eq. (4) effectively reduces to the sum of two Lorentzians,

$$A_\sigma(\omega) = \frac{\Gamma_\sigma}{\Gamma_1+\Gamma_2}\frac{1}{\pi}\frac{(\Gamma_1+\Gamma_2)/2}{(\omega-\epsilon_\sigma)^2 + \left[(\Gamma_1+\Gamma_2)/2\right]^2} + \frac{\Gamma_{\bar\sigma}}{\Gamma_1+\Gamma_2}\frac{1}{\pi}\frac{\tilde\Gamma/2}{(\omega-\tilde\epsilon_\sigma)^2 + (\tilde\Gamma/2)^2}. \quad (7)$$

It shows a broad peak with width $\Gamma_1+\Gamma_2$, and a sharper one with width $\tilde\Gamma$ (see inset of Fig. 1). For the latter peak the level splitting $\Delta\tilde\epsilon$ and the width $\tilde\Gamma$ are given by

$$\frac{\Delta\tilde\epsilon}{\Delta\epsilon} = \frac{\tilde\Gamma}{\Gamma_1+\Gamma_2} = \frac{4\Gamma_1\Gamma_2}{(\Gamma_1+\Gamma_2)^4}(\Delta\epsilon)^2\,. \quad (8)$$

This result implies a new statistics for the renormalized interlevel spacing $\tilde s = \Delta\tilde\epsilon/\Delta$ in the regime $\tilde s \ll \Gamma/\Delta \ll 1$,

$$\tilde P_o(\tilde s) = \frac{\pi}{6}\left[\frac{(\Gamma_1+\Gamma_2)^4}{4\Gamma_1\Gamma_2\Delta^2}\right]^{2/3}\tilde s^{-1/3} \quad (9)$$

$$\tilde P_u(\tilde s) = \frac{32}{3\pi^2}\frac{(\Gamma_1+\Gamma_2)^4}{4\Gamma_1\Gamma_2\Delta^2} \quad (10)$$

and a renormalized broadening $\tilde\Gamma = 4\Gamma_1\Gamma_2(\Delta\epsilon)^2/(\Gamma_1+\Gamma_2)^3$. In the orthogonal case the distribution even diverges for $\tilde s \to 0$ with integrable divergence.

Results for a wider range of energies are displayed in Figs. 2 and 3 for the renormalized level spacing as a function of the bare one and for the new level statistics, respectively, and compared to the low-energy asymptotic results (8), (9) and (10).

What is the origin of the modified $\tilde P(\tilde s)$? If we omit the tunneling part of the Hamiltonian Eq. (1) the the eigenvalues, ϵ_k and ϵ_σ, have spectral weights totally within the corresponding states k and σ. It is tempting to assume that the coupling of the level σ to the reservoir states k leads to a renormalization of the eigenvalues ϵ_σ, thus explaining the modified level statistics. But this is not the case. The crucial point is rather the renormalization of the eigenvectors. Due to the tunneling each eigenvector acquires a finite overlap with the bare dot states. What we see in Fig. 1 is the envelope of the square of this overlap. The relevant physics is, therefore, a *spectral weight renormalization* effect rather than energy renormalization.

Signatures in an absorption experiment. How can the evolution of the sharp peak be probed? In the following we discuss consequences for an absorption experiment. For a single quantum dot the absorption power of a photon with energy ω_0, accompanied with a transition from level σ to level σ', is given by

$$\int_{-\infty}^{\infty} d\,\omega\, A_\sigma\left(\omega - \frac{\omega_0}{2}\right) A_{\sigma'}\left(\omega + \frac{\omega_0}{2}\right) \times f\left(\omega - \frac{\omega_0}{2} - \mu\right)\left[1 - f\left(\omega + \frac{\omega_0}{2} - \mu\right)\right]. \quad (11)$$

Since in realistic experiments an ensemble of dots is probed, the total signal $I_{\sigma\sigma'}$ is determined by the average over all possible configurations, i.e., we have to average over the chemical potential $\int_{\hat\epsilon-\Delta/2}^{\hat\epsilon+\Delta/2}\frac{d\mu}{\Delta}\ldots \approx \int_{-\infty}^{\infty}\frac{d\mu}{\Delta}\ldots$ as well as over bare level separation $\int_0^1 ds\, P(s)\ldots$. After performing the integral over $d\mu$ we get

$$I_{\sigma\sigma'} = \frac{\omega_0}{\Delta}\frac{e^{\beta\omega_0}}{e^{\beta\omega_0}-1}D_{\sigma\sigma'} \quad (12)$$

$$D_{\sigma\sigma'} = \int_0^1 ds\, P(s) B_{\sigma\sigma'} \quad (13)$$

$$B_{\sigma\sigma'} = \int_{-\infty}^{\infty} d\,\omega A_\sigma\left(\omega - \frac{\omega_0}{2}\right) A_{\sigma'}\left(\omega + \frac{\omega_0}{2}\right). \quad (14)$$

The quantity $B_{\sigma\sigma'}$ probes the properties of the transition for a given level spacing. It accounts for the renormalization of the level spacing and width. The total signal, however, is determined by the average over all (bare) level spacings, $D_{\sigma\sigma'}$. For $\sigma = \sigma'$ the absorption is accompanied by a transition within one level and for $\sigma \neq \sigma'$ a transition between the two levels. In the following we are interested in the asymptotic behavior for $\omega_0 \to 0$, i.e., $\omega_0 \ll \Gamma \ll \Delta$.

In order to establish some reference frame we first approximate $A_\sigma(\omega)$ by (i) delta functions (which neglects the coupling of each level to the reservoir) and by (ii) Lorentzians with width Γ_σ (which neglects the indirect coupling of the levels) before (iii) we use the exact form Eq. (4).

(i) In the first case we find $D_{12} = P(\omega_0)$, i.e., $D^o_{12} = \pi^2\omega_0/2$ and $D^u_{12} = 32\omega_0^2/\pi^2$, while $D_{11} = D_{22} = D_{21} = 0$. I.e. the absorption power vanishes for $\omega_0 \to 0$.

(ii) In the second case

$$B_{12} = \frac{1}{2\pi}\frac{\Gamma_1+\Gamma_2}{(\omega_0-\Delta\epsilon)^2+(\Gamma_1+\Gamma_2)^2/4} \tag{15}$$

$$B_{21} = \frac{1}{2\pi}\frac{\Gamma_1+\Gamma_2}{(\omega_0+\Delta\epsilon)^2+(\Gamma_1+\Gamma_2)^2/4} \tag{16}$$

$$B_{\sigma\sigma} = \frac{1}{2\pi}\frac{\Gamma_\sigma}{\omega_0^2+\Gamma_\sigma^2}, \tag{17}$$

which yields for $\omega_0 = 0$

$$D^o_{12} = D^o_{21} = \frac{1}{4}\frac{\Gamma_1+\Gamma_2}{\Delta^2}\ln\frac{2\Delta}{\Gamma_1+\Gamma_2} \tag{18}$$

$$D^u_{12} = D^u_{21} = \frac{16}{\pi^3}\frac{\Gamma_1+\Gamma_2}{\Delta^2} \tag{19}$$

and $D^o_{\sigma\sigma} = 1/4\Gamma_\sigma$, $D^u_{\sigma\sigma} = 32/3\pi^3\Gamma_\sigma$. Two points are remarkable. First, due to the broadening there is always an overlap of the shifted spectral functions, i.e. even for $\omega_0 = 0$, D_{12} remains finite. Second, for systems with time reversal symmetry the $(\Delta\epsilon)^{-2}$ behavior of Eq. (15) leads to logarithmic behavior, which is cut off at low energies by the average level width $(\Gamma_1+\Gamma_2)/2$.

(iii) The renormalization of the level splitting and width due to indirect coupling of the levels to each other is fully included in the exact spectral function Eq. (4). The leading terms are

$$D^o_{12} = D^o_{21} = \frac{1}{8}\frac{\Gamma_1+\Gamma_2}{\Delta^2}\ln\frac{\Delta}{\sqrt{\omega_0\frac{(\Gamma_1+\Gamma_2)^3}{4\Gamma_1\Gamma_2}}} \tag{20}$$

$$D^u_{12} = D^u_{21} = \frac{8}{\pi^3}\frac{\Gamma_1+\Gamma_2}{\Delta^2} \tag{21}$$

and

$$D^o_{\sigma\sigma} = \frac{1}{4\Gamma_\sigma} + \frac{1}{8}\frac{\Gamma_1+\Gamma_2}{\Delta^2}\frac{\Gamma_{\bar\sigma}}{\Gamma_\sigma}\ln\frac{\Delta}{\sqrt{\omega_0\frac{(\Gamma_1+\Gamma_2)^3}{4\Gamma_1\Gamma_2}}} \tag{22}$$

$$D^u_{\sigma\sigma} = \frac{32}{3\pi^3\Gamma_\sigma} + \frac{16}{\pi^3}\frac{\Gamma_1+\Gamma_2}{\Delta^2}\frac{\Gamma_{\bar\sigma}}{\Gamma_\sigma}\,. \tag{23}$$

We note that the low-energy cutoff $\sqrt{\omega_0\frac{(\Gamma_1+\Gamma_2)^3}{4\Gamma_1\Gamma_2}}$ is frequency dependent and determines the asymptotic absorption power as can be seen in Fig. 4. In comparison to Eqs. (18) and (19) a factor 1/2 arises due to details in the spectral density.

We remark here that for a symplectic ensemble we get similar result as for the unitary case Eq. (21) and (23), $D^s_{12} = D^s_{21} = (2^{16}/3^7\pi^4)(\Gamma_1+\Gamma_2)/\Delta^2$ and $D^s_{\sigma\sigma} = 2^{18}/(5\cdot 3^6\pi^4\Gamma_\sigma) + (2^{17}/3^7\pi^4)(\Gamma_{\bar\sigma}/\Gamma_\sigma)(\Gamma_1+\Gamma_2)/\Delta^2$.

Interaction. To get a qualitative understanding for the effect on interactions we include a finite charging energy in the Hamiltonian $H \to H + Un_1n_2$. As an example we consider the degenerate case $\Delta\epsilon = 0$. A rotation within the dot subspace, as discussed above, can still decouple one new level from the reservoir. But the levels still influence each other due to the charging energy term $Un_1'n_2'$ and the conclusion that there will be a delta-function peak in the spectral density is no longer valid. If we neglect all terms in the equations of motion which would correspond to correlations in the reservoirs we get a closed set of equations for the Green's functions. The resulting expression for $G_{\sigma\sigma}(\omega+i0^+)$ for arbitrary U is too lengthy to be presented here. However, for $U\to\infty$ it simplifies to

$$G_{\sigma\sigma}(\omega) = \frac{1}{2}\left(\frac{1-\langle n\rangle}{\omega-\epsilon+i\Gamma} + \frac{1-\langle n\rangle}{\omega-\epsilon-2\hat\Sigma(\omega)}\right) \tag{24}$$

with $\mathrm{Im}\,\hat\Sigma(\omega+i0^+) = -\Gamma/2f(\omega)$ and $\mathrm{Re}\,\hat\Sigma(\omega+i0^+) = \Gamma/2\pi\,[\ln(\beta U/2\pi) - \mathrm{Re}\,\Psi(1/2+i\beta\omega/2\pi)]$. Here, $\Psi(z)$ is the digamma function and $f(\omega)$ is the Fermi function. If the levels are far (in units of Γ_σ) from the Fermi level they only contribute to the absorption power in the case $|\omega-\epsilon| \lesssim k_BT$ since otherwise the levels are totally empty or filled. Then, the second term acquires a width of the order of Γ and does not show a sharp peak. If the levels are close to the Fermi level also the low-temperature regime is important. But then the spectral density shows a richer structure related to collective many-particle states. Similar effects lead to Kondo physics in a single-level spin-degenerate quantum dots with spin conservation (see e.g. [15,16]). The analysis of the level's properties and their effect on the absorption power becomes more complicated and is out of the aim of this Letter.

In summary, our model analysis produced a modified statistics of the broadened levels due to a renormalization of the spectral weights. As a consequence, in the orthogonal case the asymptotic absorption power depends logarithmically on the external frequency.

Acknowledgments. We acknowledge useful discussions with J.L. Pichard. J.K. and Y.G. acknowledge the hospitality of the Weizmann Institute of Science and the University of Karlsruhe, respectively. This work was supported by the DFG as part of SFB 195, by the German-Israeli Foundation (GIF), by the Israel Science Foundation founded by the Israel Academy of Sciences and Humanities - Centers of Excellence Program, and by the U.S.-Israel Binational-Science Foundation (BSF).

[1] L.P. Gorkov and G.M. Eliashberg, Zh. Eksp. Teor. Fiz. **48**, 1407 (1965) Sov. Phys. JETP **21**, 940 (1965).

[2] B.I. Shklovskii, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 287 (1982) JETP Lett. **36**, 352 (1982).

[3] K.B. Efetov Adv. Phys. **32**, 53 (1983).
[4] Y. Imry Europhys. Lett. **1**, 249 (1986).
[5] A. Kamenev, B. Reulet, H. Bouchiat and Y. Gefen, Europhys. Lett. **28**, 391 (1994).
[6] K.B. Efetov, *Supersymmetry in Disorder and Chaos*, Cambridge University Press, New York (1997)
[7] C.W. Beenakker Rev. Mod. Phys. **69**, 731 (1997).
[8] T. Guhr, A. Müller-Groeling and H.A. Weidenmüller, cond-mat/9707301.
[9] M.L. Mehta, *Random Matrices*, second edition, Academic Press, (1991).
[10] A. Kamenev and Y. Gefen, Int. J. Mod. Phys. **B9**,751 (1995).
[11] This applies for the limit of $\Delta \gg \omega_0 > \Gamma$, where Γ is the level's broadening.
[12] F. Zhou, B. Spivak, N. Taniguchi, and B.L. Altshuler, Phys. Rev. Lett. **77**, 1958 (1996).
[13] J.L.Pichard and B.Shapiro, J. Physique **4**, 623 (1994).
[14] This describes either the situation in which the spin labeled by σ is not a conserved quantum number or in which σ labels some other non-conserved quantum number. In the latter case spin is ignored either due to strong Zeeman splitting or since for a non-interacting quantum dot the total Hamiltonian separates into two parts describing either spin. The corresponding levels do not influence each other.
[15] J. König, H. Schoeller, and G. Schön, Phys. Rev. Lett. **76**, 1715 (1996).
[16] J. König, J. Schmid, H. Schoeller, and G. Schön, Phys. Rev. B **54**, 16820 (1996).

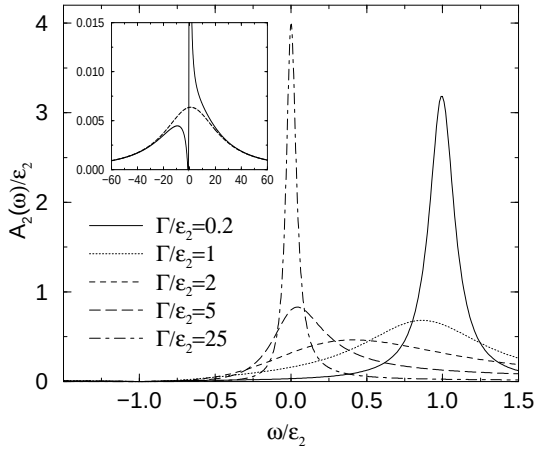

FIG. 1. Spectral density for level 2 with $\Gamma_1 = \Gamma_2 = \Gamma$ and $\epsilon_1 = -\epsilon_2 = 1$. For $\Gamma \ll \epsilon$ a peak with width $\tilde{\Gamma} \ll \Gamma$ evolves near zero. The spectral density of level 1 is related to that of level 2 by $A_1(\omega) = A_2(-\omega)$. Inset: Spectral density for $\Gamma/\epsilon_2 = 25$ (solid line) showing a sharp peak sitting atop a Lorentzian with width 2Γ (dashed line).

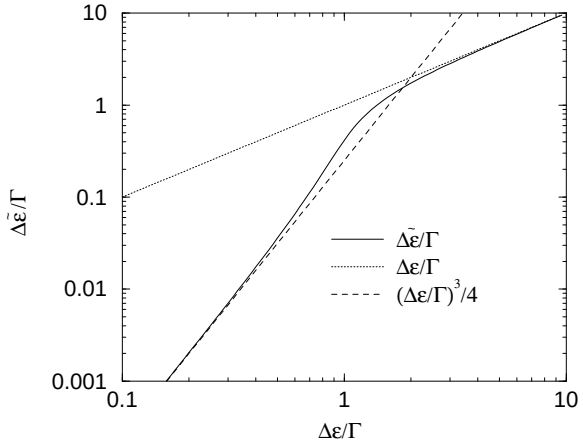

FIG. 2. Renormalized level spacing as a function of the bare one in double logarithmic plot for $\Gamma_1 = \Gamma_2 = \Gamma$. The dotted and the dashed curve are the high- and low-energy (see Eq. (8)) asymptotes, respectively.

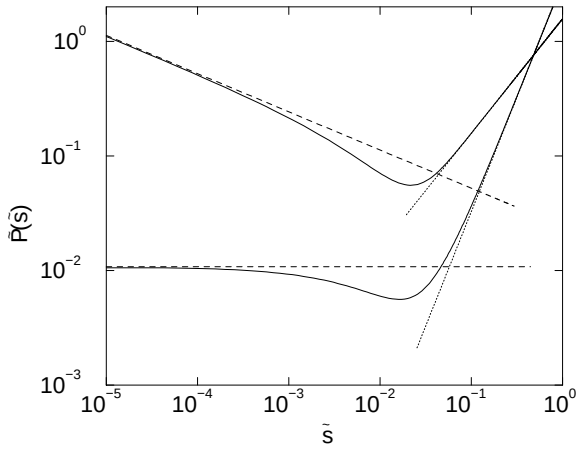

FIG. 3. Solid lines: new level statistics $\tilde{P}(\tilde{s})$ in the orthogonal (upper curve) and unitary ensemble (lower curve) with $\Gamma_1 = \Gamma_2 = \Gamma = \Delta/20$. Dashed lines: low-energy asymptotes Eqs. (9) and (10). Dotted lines: high-energy asymptotes (Breit-Wigner distribution).

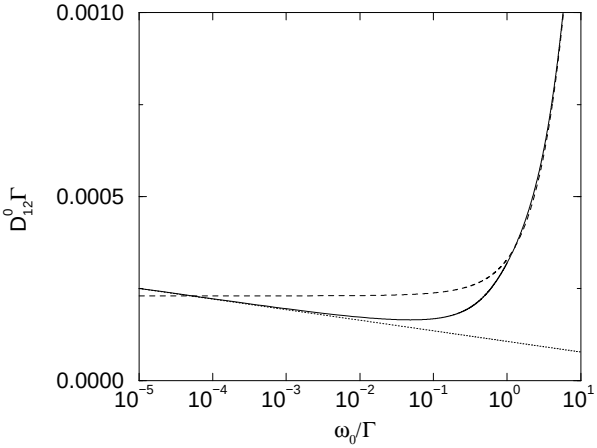

FIG. 4. For the orthogonal ensemble the asymptotic absorption power is logarithmically divergent at small frequencies (solid line). In the simplified model where the indirect coupling between the levels is neglected, the low-energy behavior shows a saturation at Γ (dashed line). The dotted curve is the asymptote Eq. (20).